Synchronization of non-identical chaotic systems: an exponential dichotomies approach

2001 J. Phys. A: Math. Gen. 34 9143

(http://iopscience.iop.org/0305-4470/34/43/304)

J. Phys. A: Math. Gen. **34** (2001) 9143–9151 PII: S0305-4470(01)26700-6

Synchronization of non-identical chaotic systems: an exponential dichotomies approach

A Acosta[1] and P García[2]

[1] Departamento de Matemática Aplicada, Facultad de Ingeniería, Universidad Central de Venezuela, AP 48110, Caracas, 1041-A, Venezuela
[2] Departamento de Física Aplicada, Facultad de Ingeniería, Universidad Central de Venezuela, AP 48110, Caracas, 1041-A, Venezuela

E-mail: pedro@apollo.ciens.ucv.ve

Received 5 July 2001, in final form 29 August 2001
Published 19 October 2001
Online at stacks.iop.org/JPhysA/34/9143

Abstract
In most applications, the synchronization of systems evolving under a chaotic regime requires the construction of identical systems or subsystems. In practical applications, systems should be created so that they match as closely as possible. Moreover, in real devices parameters can fluctuate resulting in loss of synchronization. In this paper, we consider a master–slave system of ordinary differential equations which are not identical. Considering bounded solutions of the master equation, we use those as an input in the slave equation. By using exponential dichotomies techniques we establish conditions that ensure synchronization.

PACS numbers: 05.45.+b, 02.30.Hq

1. Introduction

Synchronization between chaotic dynamical systems has been an active research topic since it was introduced by Fujisaka and Yamada [1]. The reason for this increasing interest is the wide range of applications which go from communication [2–5] to biology [6, 7]. The synchronization of chaotic systems can be achieved in several forms, for example, the work of Pecora and Caroll [8] shows that under suitable conditions, two chaotic systems S_1 and S_2 can be synchronized if S_2 is formed copying a subsystem that is a replica of part of the system S_1. Another possibility consists in coupling S_1 and S_2 by a small linear term, in which the difference between the current state of the two systems is used as an inhibitory effect on the separation of the orbits [9].

A common feature of this and other methodologies is that the considered systems are identical. In practical applications, it is impossible to construct devices with identical parameters. Therefore, if we want to synchronize real systems it seems to be more convenient to consider models with different parameters.

In this paper, we study master–slave non-identical systems, where master–slave means that one of the systems (master) evolves freely while the other (slave) is driven by the master. The class of systems that we consider satisfies the conditions so that the parameters, which are not identical, appear to affect only linear terms. Systems illustrating this class are Lorenz and Rossler equations.

In order to obtain synchronization between master–slave non-identical systems, we proceed as follows. First, a bounded solution of the master equation is considered and it is used as an input in the slave equation. Next, our attention is focused on the non-autonomous system that is obtained from the slave equation and, by using exponential dichotomies theory in an appropriate framework, we establish conditions that ensure synchronization of our master–slave system.

The rest of this paper is organized as follows. In section 2, we set our problem and present what is needed regarding exponential dichotomies. Section 3 is devoted to proving the main result and in section 4, this result is applied to a particular example. Finally, in section 5 we give some concluding remarks.

2. Setting of the problem and exponential dichotomies

We consider the system

$$\dot{x} = f(\bar{\mu}, x) \tag{1}$$

$$\dot{y} = f(\mu, y) + \nu(x - y) \tag{2}$$

where ν is a real constant and $f\colon \Re^m \times \Re^n \to \Re^n$ is a continuous function that satisfies the following hypotheses:

H1 $f(\mu, z) = B(\mu)z + g(z)$, where B is a matrix of dimension $n \times n$ that depends on the vector parameter μ and g is a non-linear function.

H2 $f(\mu, z + x) - f(\mu, x) = f(\mu, z) + C(x)z$, where C is a matrix of dimension $n \times n$ that depends on x.

H3 There exists $K_1 > 0$ so that

$$|f(\mu, z) - f(\bar{\mu}, z)| \leqslant K_1 |\mu - \bar{\mu}|\,|z|.$$

Also, we assume for the nonlinear function g that

H4 $|g(z) - g(w)| \leqslant \eta(\rho)|z - w|\,|z|$, for all $z, w \in \Re^n$ such that $|z|, |w| \leqslant \rho$, where η is a continuous, non-decreasing, non-negative function on $[0, \infty)$ with $\eta(0) = 0$.

In addition to the previous hypothesis it is important to remark that throughout this work we assume $\bar{\mu}$ and μ as constant vectors.

We define the set B_m as

$$B_m := \left\{x_0 \in \Re^n : x(t, x_0, \bar{\mu}) \text{ is bounded on } [0, \infty)\right\}. \tag{3}$$

Now let $x(t, x_0, \bar{\mu})$ denote a solution of equation (1) (master) satisfying $x(0, x_0, \bar{\mu}) = x_0$ and consider it as an input in equation (2) (slave) for which $y(t, x_0, y_0, \mu, \bar{\mu})$ denotes the solution satisfying $y(0, x_0, y_0, \mu, \bar{\mu}) = y_0$.

Definition 2.1. *Let $x_0 \in B_m$, we say that the system (1)–(2) synchronizes along the trajectory $x(t, x_0, \bar{\mu})$, $t \geqslant 0$, if there exists a set V in $\Re^n$ such that: if given $\epsilon > 0$, then $\delta > 0$ exists such that if $|\mu - \bar{\mu}| < \delta$ and $y_0 - x_0 \in V$, then*

$$\lim_{t\to\infty} \sup |y(t, x_0, y_0, \mu, \bar{\mu}) - x(t, x_0, \bar{\mu})| < \epsilon.$$

We now consider some properties of exponential dichotomies of linear systems of differential equations. We present some lemmas to be applied in the next two sections.

Let $A : J \to \Re^{n\times n}$ be continuous, where J is some interval, and consider the differential equation:

$$\dot{z} = A(t)z \tag{4}$$

Let $\Phi(t, s)$, $\Phi(t, t) = I$, be the principal matrix solution of (4).

Definition 2.2. *We say that (4) has an exponential dichotomy on the interval J if there are projections $P(t)$: $\Re^n \to \Re^n$, $t \in J$, continuous in t, such that if $Q(t) := I - P(t)$, where I is the identity matrix, then:*

(i) $\Phi(t, s)P(s) = P(t)\Phi(t, s), t, s \in J.$
(ii) $|\Phi(t, s)P(s)| \leqslant K\mathrm{e}^{-\alpha(t-s)}, t \geqslant s \in J.$
(iii) $|\Phi(t, s)Q(s)| \leqslant K\mathrm{e}^{\alpha(t-s)}, s \geqslant t \in J.$

where K and α are positive constants.

The two cases of most interest are where J is the positive half-line $[0, \infty)$ and the whole line $\Re$. However, we are only interested in the first case.

In the case of the autonomous equation

$$\dot{z} = A_0 z$$

there is an exponential dichotomy on $[0, \infty)$ if and only if no eigenvalue of the constant matrix A_0 has zero real part. In this example associated with the trivial solution there are two sets called the stable manifold and the unstable manifold. The concept of exponential dichotomy provides the notions of those sets for the non-autonomous equations. Consider the inhomogeneous equation

$$\dot{z} = A(t)z + f(t) \tag{5}$$

where f is in the Banach space of all bounded continuous functions with the supremun norm.

Lemma 2.1. *Suppose that (4) has an exponential dichotomy on* $[0, +\infty)$*. For any solution $z(t)$ of (5) which exists and is bounded on* $[0, +\infty)$*, there is an $z_0 \in$ Range of $P(0)$ such that $z(t)$ satisfies*

$$z(t) = \Phi(t, 0)z_0 + \int_0^t P(t)\Phi(t, s)f(s)\,\mathrm{d}s + \int_\infty^t Q(t)\Phi(t, s)f(s)\,\mathrm{d}s \qquad t \geqslant 0. \tag{6}$$

Conversely, any solution of (6) bounded on $[0, +\infty)$ *is a solution of (6).*

Proof. See [10]. □

Now, we consider a perturbation of the differential equation (4). Let $B : [0, +\infty) \to \Re^{n\times n}$ be a bounded, continuous matrix function.

Lemma 2.2. *Suppose that* (4) *has an exponential dichotomy on* $[0, +\infty)$*. If $\delta := \sup|B(t)| < \alpha/4K^2$, then the perturbed equation*

$$\dot{z} = (A(t) + B(t))z \tag{7}$$

also has an exponential dichotomy on $[0, +\infty)$ *with constants $\tilde{K}$ and $\tilde{\alpha}$ determined by K, α and δ. Moreover if $\tilde{P}(t)$ is the corresponding projection, then $|P(t) - \tilde{P}(t)| = O(\delta)$ uniformly in $t \in [0, +\infty)$. Also $|\tilde{\alpha} - \alpha| = O(\delta)$.*

Proof. See [10, 11]. □

To set up the problem in a framework where exponential dichotomies can be applied, we consider, for $x_0 \in B_m$, the following transformation of variables

$$z = y - x(t, x_0, \bar{\mu}). \tag{8}$$

If y is a solution of the slave equation with input $x(t, x_0, \bar{\mu})$, then the transformation (8) applied to this equation yields the equation

$$\dot{z} = A(\nu, \mu, x(t, x_0, \bar{\mu}))z + F(\mu, z, x(t, x_0, \bar{\mu})) \tag{9}$$

where

$$A(\nu, \mu, x(t, x_0, \bar{\mu})) := \nu I + B(\mu) + C(x(t, x_0, \bar{\mu})) \tag{10}$$

and

$$F(\mu, z, x(t, x_0, \bar{\mu})) := g(z) + f(\mu, z, x(t, x_0, \bar{\mu})) - f(\bar{\mu}, z, x(t, x_0, \bar{\mu})). \tag{11}$$

We will assume, in the next section, that the linear equation corresponding to (10), i.e.

$$\dot{z} = A(\nu, \mu, x(t, x_0, \bar{\mu}))z \tag{12}$$

has an exponential dichotomy on $[0, +\infty)$.

3. Main result

Two lemmas, one on the existence of solutions of equation (9) and another which is related to the Gronwall inequality, will be the key elements to establish our main result.

From now on we assume that equation (12) has an exponential dichotomy on $[0, +\infty)$ with projections $P(t)$ and where α and K are the corresponding constants. Also, we assume that $x_0 \in B_m$.

Let $\rho > 0$ and $\mu \in \Re^m$ such that

$$\eta(\rho) < \frac{\alpha}{8K} \tag{13}$$

$$|\mu - \bar{\mu}| < \frac{\alpha\rho}{8KK_1 \sup_{t\geqslant 0}|x(t, x_0, \bar{\mu})|}. \tag{14}$$

With this choice of ρ and μ, and for any z_0 in the range of $P(0)$ with $|z_0| < \rho/2K$, we define $\mathcal{G}(z_0, \rho, \nu, \mu)$ as a set of continuous functions $z : [0, +\infty) \to \Re^n$ such that $|z| := \sup_{t\geqslant 0}|z(t)| \leqslant \rho$ and $P(0)z(0) = z_0$. $\mathcal{G}(z_0, \rho, \nu, \mu)$ is a closed bounded subset of the Banach space of all continuous functions taking $[0, +\infty)$ into $\Re^n$ with uniform topology. For any $z \in \mathcal{G}(z_0, \rho, \nu, \mu)$, we define Tz by

$$\begin{aligned}(Tz)(t) = \Phi(t, 0)z_0 &+ \int_0^t P(t)\Phi(t, s)F(\mu, z(s), x(s, x_0, \bar{\mu}))\,\mathrm{d}s \\ &+ \int_\infty^t Q(t)\Phi(t, s)F(\mu, z(s), x(s, x_0, \bar{\mu}))\,\mathrm{d}s \qquad t \geqslant 0.\end{aligned}$$

Lemma 3.1. *T acts from $\mathcal{G}(z_0, \rho, \nu, \mu)$ into itself and also has a unique fixed point in $\mathcal{G}(z_0, \rho, \nu, \mu)$.*

Proof. Given $\mathcal{G}(z_0, \rho, \nu, \mu)$, it is easy to see that Tz is defined and continuous for $t \geqslant 0$ with $P(0)(Tz)(0) = z_0$.

The fact that (12) has an exponential dichotomy on $[0, \infty)$ and the definition of F produce, for $t \geqslant 0$, the estimation

$$|(Tz)(t)| \leqslant K\,\mathrm{e}^{-\alpha t}|z_0| + \int_0^t K\,\mathrm{e}^{-\alpha(t-s)}|g(z(s))|\,\mathrm{d}s + \int_0^t K\,\mathrm{e}^{-\alpha(t-s)}|f(\mu, x(s)) - f(\bar{\mu}, x(s))|\,\mathrm{d}s + \int_t^\infty K\,\mathrm{e}^{\alpha(t-s)}|g(z(s))|\,\mathrm{d}s + \int_t^\infty K\,\mathrm{e}^{\alpha(t-s)}|f(\mu, x(s)) - f(\bar{\mu}, x(s))|\,\mathrm{d}s$$

where $x(s) \equiv x(s, x_0, \bar{\mu})$. Now, since $g(0) = 0$ and from H3, H4, we obtain

$$|g(z(s))| + f(\mu, x) - |f(\bar{\mu}, x)| \leqslant \eta(\rho)\rho + K_1|\mu - \bar{\mu}| \sup_{\tilde{s}\geqslant 0}|x(\tilde{s})|.$$

Therefore,

$$|Tz(t)| \leqslant K\,\mathrm{e}^{-\alpha t}|z_0| + K\left(\eta(\rho)\rho + K_1|\mu - \bar{\mu}| \sup_{\tilde{s}\geqslant 0}|x(\tilde{s})|\right)\left(\int_0^t \mathrm{e}^{-\alpha(t-s)}\,\mathrm{d}s + \int_t^\infty \mathrm{e}^{\alpha(t-s)}\,\mathrm{d}s\right)$$
$$\leqslant K|z_0| + \frac{2K}{\alpha}\left(\eta(\rho)\rho + K_1|\mu - \bar{\mu}| \sup_{\tilde{s}\geqslant 0}|x(\tilde{s}|\right).$$

Thus, from (14) and the condition $|z_0| < \rho/2K$, we obtain $|Tz| < \rho$. Therefore, T acts from $\mathcal{G}(z_0, \rho, \nu, \mu)$ into itself.

Furthermore, the same types of estimates yield, for z and $w \in \mathcal{G}(z_0, \rho, \nu, \mu)$,

$$|(Tz)(t) - (Tw)(t)| \leqslant \frac{2K}{\alpha}\eta(\rho)|z - w| \leqslant \frac{1}{4}|z - w| \qquad \text{for} \quad t \geqslant 0.$$

Thus, T is a contraction on $\mathcal{G}(z_0, \rho, \nu, \mu)$ and it has a unique fixed point. □

Lemma 3.2. *Suppose $a > 0, b > 0, \mathcal{K}, \mathcal{L}, \mathcal{M}$ are non-negative constants and u is a non-negative bounded continuous solution of the inequality*

$$u(t) \leqslant \mathcal{K}\,\mathrm{e}^{-at} + \mathcal{L}\int_0^t \mathrm{e}^{-a(t-s)}u(s)\,\mathrm{d}s + \mathcal{M}\int_t^\infty \mathrm{e}^{b(t-s)}u(s)\,\mathrm{d}s \qquad t \geqslant 0.$$

If

$$\beta := \frac{\mathcal{L}}{a} + \frac{\mathcal{M}}{b} < 1$$

then

$$u(t) \leqslant (1 - \beta)^{-1}\mathcal{K}\,\mathrm{e}^{-[a-(1-\beta)^{-1}\mathcal{L}]t}.$$

Proof. See [12]. □

Let $z^*(\cdot, z_0, \nu, \mu)$ denote the fixed point in lemma 3.1. An important remark is that using the same estimates as above, one shows that the function $z^*(\cdot, z_0, \nu, \mu)$ is continuous on the variables z_0 and μ and $z^*(\cdot, 0, \nu, \mu) = 0$.

Our main result, which implies that the master–slave system synchronizes, is presented now.

Theorem 3.1. *If the hypotheses H1, H2, H3 and H4 are satisfied and equation (12) has an exponential dichotomy on* $[0, +\infty)$*, then under the estimates (13) and (14),* $z^*(\cdot, z_0, \nu, \mu)$ *satisfies the estimation*

$$|z^*(t, z_0, \nu, \mu)| \leqslant \frac{4}{3} K|z_0|\,\mathrm{e}^{-\frac{5}{6}\alpha t} + \frac{1}{4}|z^*(\cdot, 0, \nu, \mu)| + \frac{2KK_1}{\alpha}|\mu - \bar{\mu}||x(\cdot, x_0, \bar{\mu})| \qquad t \geqslant 0. \tag{15}$$

Proof. Let $z^*_{z_0}(t)$ denote the fixed point $z^*_{z_0}(t, z_0, \nu, \mu)$.

First, estimations for $z^*_{z_0}$ and the difference between the fixed points $z^*_{z_0}$ and $z^*_{\tilde{z}_0}$ are obtained.

$$\begin{aligned}
|z^*_0(t)| &\leqslant \int_0^t K\,\mathrm{e}^{-\alpha(t-s)}|g(z^*_0(s))|\,\mathrm{d}s + \int_t^\infty \mathrm{e}^{\alpha(t-s)}|g(z^*_0(s))|\,\mathrm{d}s \\
&\quad + \int_0^t K\,\mathrm{e}^{-\alpha(t-s)}|f(\mu, x(s)) - f(\bar{\mu}, x(s))|\,\mathrm{d}s \\
&\quad + \int_t^\infty \mathrm{e}^{\alpha(t-s)}|f(\mu, x(s)) - f(\bar{\mu}, x(s))|\,\mathrm{d}s \\
&\leqslant K\eta(\rho)|z^*_0(\cdot)| \left\{ \int_0^t \mathrm{e}^{-\alpha(t-s)}\,\mathrm{d}s + \int_t^\infty \mathrm{e}^{\alpha(t-s)}\,\mathrm{d}s \right\} \\
&\quad + KK_1|\mu - \bar{\mu}||x(\cdot)| \left\{ \int_0^t \mathrm{e}^{-\alpha(t-s)}\,\mathrm{d}s + \int_t^\infty \mathrm{e}^{\alpha(t-s)}\,\mathrm{d}s \right\} \\
&\leqslant \left\{ \frac{\alpha}{8}|z^*_0(\cdot)| + KK_1|\mu - \bar{\mu}||x(\cdot)| \right\} \frac{2}{\alpha} \\
&= \frac{1}{4}|z^*_0(\cdot)| + \frac{2KK_1}{\alpha}|\mu - \bar{\mu}||x(\cdot)|
\end{aligned}$$

$$\begin{aligned}
\left|z^*_{z_0}(t) - z^*_{\tilde{z}_0}(t)\right| &\leqslant K\,\mathrm{e}^{-\alpha t}|z_0 - \tilde{z}_0| + \frac{\alpha}{8}\int_0^t K\,\mathrm{e}^{-\alpha(t-s)}|z^*_{z_0}(s) - z^*_{\tilde{z}_0}(s)|\,\mathrm{d}s \\
&\quad + \frac{\alpha}{8}\int_0^t K\,\mathrm{e}^{\alpha(t-s)}\left|z^*_{z_0}(s) - z^*_{\tilde{z}_0}(s)\right|\,\mathrm{d}s.
\end{aligned}$$

If lemma 3.2 is applied with $a = b = \alpha$, $\mathcal{K} = K|z_0 - \tilde{z}_0|$, $\mathcal{L} = \mathcal{M} = \alpha/8$, then $\beta = 1/4$ and

$$\left|z^*_{z_0}(t) - z^*_{\tilde{z}_0}(t)\right| \leqslant \frac{4}{3}K|z_0 - \tilde{z}_0|\,\mathrm{e}^{-\frac{5}{6}\alpha t}.$$

Finally, we apply the previous estimations, with $\tilde{z}_0 = 0$, to the right-hand side of the following inequality:

$$\left|z^*_{z_0}(t)\right| \leqslant \left|z^*_{z_0}(t) - z^*_0(t)\right| + |z^*_0(t)|.$$

□

4. Application

In order to apply our main result we use the Lorenz equations:

$$\begin{aligned}
\dot{x}_1 &= \bar{\sigma}(y_1 - x_1) \\
\dot{y}_1 &= \bar{r}x_1 - y_1 - x_1 z_1 \\
\dot{z}_1 &= x_1 y_1 - \bar{b}z_1.
\end{aligned}$$

It satisfies hypotheses *H1*, *H2*, *H3* and *H4*, and the master–slave system results in

$$
\begin{aligned}
\dot{x}_1 &= \bar{\sigma}(y_1 - x_1)\\
\dot{y}_1 &= \bar{r}x_1 - y_1 - x_1 z_1\\
\dot{z}_1 &= x_1 y_1 - \bar{b}z_1\\
\dot{x}_2 &= \sigma(y_2 - x_2) + \nu(x_2 - x_1)\\
\dot{y}_2 &= r x_2 - y_2 - x_2 z_2 + \nu(y_2 - y_1)\\
\dot{z}_2 &= x_2 y_2 - b z_2 + \nu(z_2 - z_1).
\end{aligned}
$$

We concentrate our attention in the master–slave system with the usual parameters $\bar{\sigma} = 10$, $\bar{r} = 28$ and $\bar{b} = 8/3$. In this case let $(x_1(t), y_1(t), z_1(t))$ be a bounded solution of the master equation. In this particular case, the matrix

$$A(\nu, \mu, x(t, x_0, \bar{\mu})) := \nu I + B(\mu) + C(x(t, x_0, \bar{\mu}))$$

where $\mu = (\sigma, r, b)$, $\bar{\mu} = (10, 28, 8/3)$, $x_0 = (x_1(0), y_1(0), z_1(0))$ and $x(t, x_0, \bar{\mu}) = (x_1(t, x_0, \bar{\mu}), y_1(t, x_0, \bar{\mu}), z_1(t, x_0, \bar{\mu}))$, is given by

$$
\begin{pmatrix} \nu & 0 & 0\\ 0 & \nu & 0\\ 0 & 0 & \nu \end{pmatrix} + \begin{pmatrix} -\sigma & \sigma & 0\\ r & -1 & 0\\ 0 & 0 & -b \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0\\ -z_1(t) & 0 & -x_1(t)\\ y_1(t) & x_1(t) & 0 \end{pmatrix}.
$$

The eigenvalues of the matrix

$$
\begin{pmatrix} \nu - \sigma & \sigma & 0\\ r & \nu - 1 & 0\\ 0 & 0 & \nu - b \end{pmatrix}
$$

are

$$
\begin{aligned}
\lambda_1 &= \nu - b\\
\lambda_2 &= \frac{2\nu - \sigma - 1 + [(\sigma + 1)^2 + 4\sigma(r - 1)]^{1/2}}{2}\\
\lambda_3 &= \frac{2\nu - \sigma - 1 - [(\sigma + 1)^2 + 4\sigma(r - 1)]^{1/2}}{2}.
\end{aligned}
$$

In particular, for $\sigma = 10$, $r = 28$ and $b = 8/3$, the eigenvalues are

$$
\begin{aligned}
\lambda_1 &= \nu - \tfrac{8}{3}\\
\lambda_{2,3} &= \frac{2\nu - 11 \pm \sqrt{1201}}{2}.
\end{aligned}
$$

For $\nu < (11 - \sqrt{1201})/2$, these eigenvalues are negative and the system

$$\dot{z} = (\nu I + B(\bar{\mu}))z$$

has an exponential dichotomy on $[0, \infty)$ with $P = \textit{identity}$, $K = 1$ and $\alpha = (11 - 2\nu - \sqrt{1201})/2$.

For small deviations of the classical parameters, i.e '$\mu - \bar{\mu}$ small', we have that for the interval $(-\infty, \nu_0)$ with ν_0 close to $(11 - \sqrt{1201})/2$ all the eigenvalues of the matrix $\nu I + B(\mu)$ are negative and the system

$$\dot{z} = (\nu I + B(\mu))z$$

has an exponential dichotomy on $[0, \infty)$ with $P = \textit{identity}$, $K = 1$ and α close to $(11 - 2\nu - \sqrt{1201})/2$.

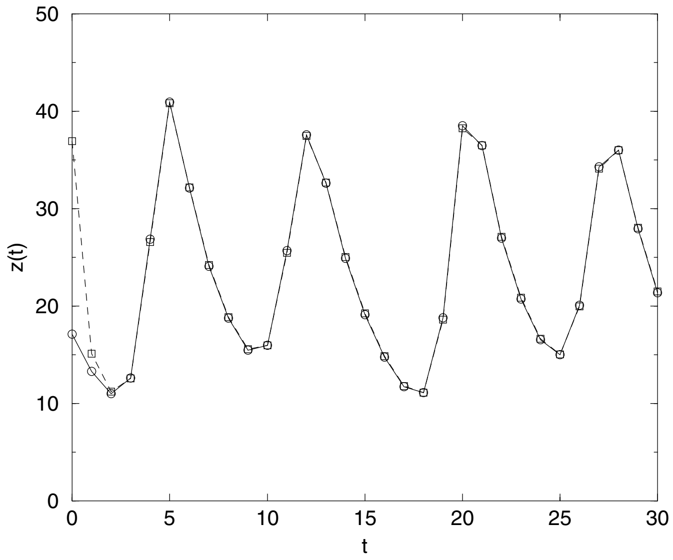

Figure 1. Synchronization of z-coordinates.

Now if $\sup|C(x(t, x_0, \bar{\mu}))| < \alpha/4$, then from lemma 2.2, we obtain that the system

$$\dot{z} = (\nu I + B(\mu) + C(x(t, x_0, \bar{\mu})))z$$

has an exponential dichotomy on $[0, \infty)$.

In the simulation shown in figure 1 we have selected $\mu = (9.8, 28.2, 2.56)$ and $\nu = -23$. We observe in the figure the evolution of the z-coordinate on the master equation and also in the slave equation.

5. Conclusions

In order to establish synchronization of non-identical chaotic systems we have presented an approach based on the theory of exponential dichotomies.

This approach also allows the following:

- An estimation, in order to achieve control over a given chaotic system, of the intensity of the necessary perturbation to maintain the orbit of the slave system on the given orbit of the master system.
- An estimation of the robustness of the synchronization against fluctuations in the parameter space around the given parameters in the master–slave system.

Acknowledgment

This research was partially supported by Consejo de Desarrollo Científico y Humanístico, Universidad Central de Venezuela.

References

[1] Fujisaka H and Yamada T 1983 *Prog. Theor. Phys.* **69** 32
[2] Kokarev L and Parlitz U 1995 *Phys. Rev. Lett.* **74** 528

[3] Gregory D, Van Wiggeren and Rajarshi Roy 1998 *Phys. Rev. Lett.* **87** 3547
[4] Rong He and Vaidya P G 1998 *Phys. Rev.* E **57** 1532
[5] Rodrigues H M 1996 *Appl. Anal.* **62** 263
[6] Glass L and Mackey M 1988 *From Clock to Chaos: The Rhythms of Life* (Princeton, NJ: Princeton University Press)
[7] Mirollo R and Strogatz S 1990 *SIAM J. Appl. Math.* **50** 1645
[8] Pecora L M and Caroll T L 1990 *Phys. Rev. Lett.* **64** 821
[9] Parmaranda P 1998 *Phys. Lett.* A **240** 55
[10] Palmer K J 1978 *J. Diff. Eqns* **55** 225
[11] Coppel W A 1978 *Dichotomies in Stability Theory (Lecture Notes in Mathematics)* vol 629 (New York: Springer)
[12] Hale J 1980 *Ordinary Differential Equations* (New York: Kriever)